Level-Set Based Topology Optimization Using Remeshing Techniques for Magnetic Actuator Design

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This paper proposes a new level-set based topology optimization method for magnetic actuator design using remeshing techniques that can generate meshes on the exact structural boundaries to improve the accuracy of finite element analysis. Two remeshing techniques: the adaptive mesh method for the first-order triangular elements and the extended finite element method (XFEM) for the first-order quadrilateral elements were employed for the optimization process. To control the computational time with the economical meshing and the accuracy of the analysis, a new scheme that can manage the level of mesh density around the level-set boundaries is proposed for the adaptive mesh method. The optimization problem was formulated to maximize the magnetic force between the core and an armature under the volume constraint of the ferromagnetic material. To verify the effectiveness of the proposed method, it was applied to an electromagnetic problem for optimal C-core actuator design that is very sensitive to structural boundaries.

***Index Terms*— adaptive mesh, extended finite element method, level-set based topology optimization, magnetic actuator design**

I. INTRODUCTION

A level-set based topology optimization method for the optimal design of magnetic devices has been successfully applied due to its clear boundary expression compared to the element based topology optimization method [1]. Since the conventional level-set method is based on the Eulerian concept in which the meshes are fixed in the design domain, the level-set boundaries of the structure cannot be precisely represented by finite elements, resulting in a decrease in the accuracy of analysis. In electromagnetic fields, it is very important to consider the exact material boundaries since the magnetic force for the actuator design is calculated by integrating Maxwell's stress tensor over the exterior surfaces of structure [2]-[3]. To overcome this problem, remeshing techniques such as the adaptive mesh method [4] and the extended finite element method (XFEM) [5] are considered in order to modify the structural boundaries more effectively.

This paper proposes a new level-set based topology optimization method for the optimal design of magnetic devices by employing two remeshing techniques that are able to consider the first-order triangular and the quadrilateral elements, respectively. The mesh coincides with the material boundaries by tracking the zero level-set and re-generating the finite elements during the optimization process. A modified adaptive remeshing technique for the triangular elements is proposed to control the level of mesh density by introducing a resolution parameter that can regulate the number of boundary points. To update the level-set functions after remeshing, a linear interpolation scheme is applied to the triangular elements and the homography estimation method used in image processing [6] is employed for the quadrilateral elements. To verify the effectiveness of the proposed method, it is applied to an electromagnetic problem of a C-core actuator that is very sensitive to structural boundaries. It was found that the proposed method can obtain an improved value of the design objective function compared to the conventional method, which uses a fixed mesh.

II. REMESHING ALGORITHM

A. Calculation of the level-set boundary points

Fig. 1 shows the boundary elements of the first-order triangular and the quadrilateral elements.

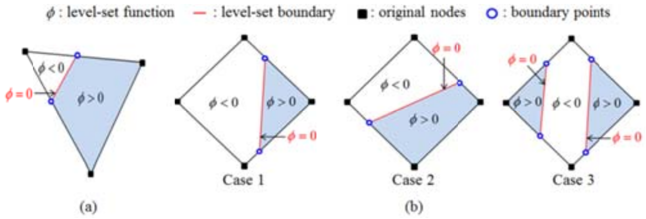


Fig. 1. Boundary expression: (a) triangular element, (b) quadrilateral element.

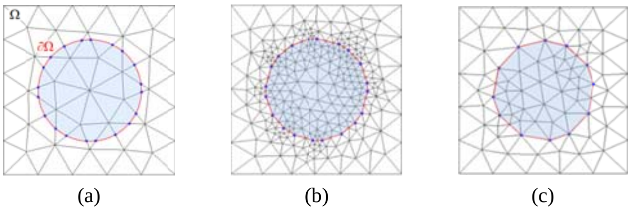
To calculate the boundary points on the triangular and the quadrilateral element, the linear interpolation and the isoparametric mapping scheme were employed, respectively.

B. Resolution control of the boundary points

The resolution parameter (d_{cutoff}) was introduced to manage the level of mesh density around the level-set boundaries as follows:

$$d_{cutoff} = R \times l_{\min} \tag{1}$$

where R is the user defined resolution ratio multiplied by the minimum length ($l_{\min}$) of the element edges in the initial mesh. It can control the number of elements around the



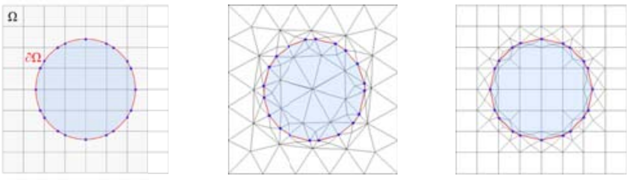
(a) (b) (c)

Fig. 2. Illustration of the resolution control: (a) fixed mesh, (b) modified adaptive mesh at R = 0.1, (c) modified adaptive mesh at R = 0.5.

level-set boundaries, as shown in Fig. 2, by eliminating the boundary points that have a distance between the two points that is greater than d_{cutoff} .

C. Update of the level-set functions

To update the level-set functions after remeshing, linear interpolation was still used for the triangular element, and homography estimation was employed for the quadrilateral element since isoparametric mapping is used to calculate the boundary points. Fig. 3 illustrates the update of the level-set functions using the homography estimation with the XFEM.



(a) (b) (c)

Fig. 3. Illustration of the XFEM: (a) fixed mesh, (b) XFEM for the triangular mesh, (c) XFEM for the quadrilateral mesh.

III. NUMERICAL EXAMPLE

The level-set based topology optimization method to maximize the magnetic force under the volume fraction (VF) for the optimal magnetic actuator design is formulated as follows:

$$\begin{aligned} &\text{maximize } F = \int_{\Omega} \mathbf{T}(\mathbf{x}) \cdot \mathbf{n} H(\phi(\mathbf{x})) d\Omega \\ &\text{subject to } G = \int_{\Omega} H(\phi(\mathbf{x})) d\Omega \Big/ \int_{\Omega} d\Omega \text{ -VF} \leq 0 \end{aligned} \tag{2}$$

where $\mathbf{T}(\mathbf{x})$ is Maxwell's stress tensor over the exterior surfaces and $\mathbf{n}$ is the normal vector at $\mathbf{x}$, and $H(\phi(\mathbf{x}))$ is the Heaviside function to consider the material properties.

To verify the effectiveness, the proposed method is applied for the optimal C-core actuator design. Fig. 4 shows the symmetric expression of the design domain. The relative permeability of the ferromagnetic material and air were set to 10000/3 and 1, respectively, and the applied current density was $1\ \text{A/mm}^2$. Fig. 5 and Fig. 6 show the optimal shapes obtained with the finite element mesh using a fixed mesh, modified adaptive remeshing technique applying a different resolution ratio (*R*), and XFEM under a volume fraction of 0.6. Table I summarizes the results, and shows that the proposed method is very effective for magnetic actuator design compared to the fixed mesh since it can obtain more improved value of design objective function with the economical meshing.

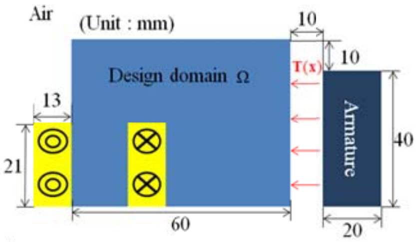
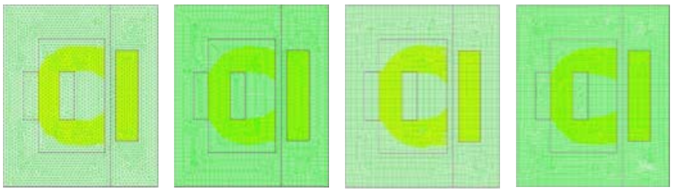
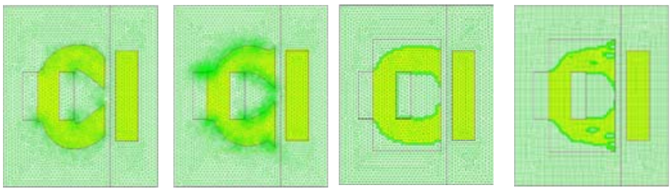


Fig. 4. Symmetric expression for the design domain of the C-core actuator



(a) (b) (c) (d)

Fig. 5. Optimal design obtained using the fixed mesh: (a) case 1(TRIA,coarse) (b) case 2(TRIA,fine), (c) case 3(QUAD,coarse), (d) case 4(QUAD,fine).



(a) (b) (c) (d)

Fig. 6. Optimal design obtained using the remeshing techniques: (a) Adaptive mesh(*R*=2), (b) Adaptive mesh(*R*=1), (c) XFEM(TRIA), (d) XFEM(QUAD).

TABLE I
MAGNETIC FORCE OF THE OPTIMAL RESULTS

	Number of Elements	Magnetic Force [N/m]
Fixed Mesh (case1)	5776	35.5
Fixed Mesh (case 2)	15786	40.5
Fixed Mesh (case 3)	6518	36.1
Fixed Mesh (case 4)	14687	39.5
Adaptive Mesh (*R*=2, case1)	7419	40.6
Adaptive Mesh (*R*=1, case1)	10915	42.6
XFEM(TRIA, case3)	6840	40.8
XFEM(QUAD, case3)	9076	42.4

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